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A note on supergravity duals of non-commutative Yang-Mills theory

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Abstract: A class of supergravity backgrounds have been proposed as dual descriptions of strong coupling large-N non-commutative Yang-Mills (NCYM) theories in $3+1$ dimensions. However calculations of correlation functions in supergravity from an evaluation of relevant classical actions appear ambiguous. We propose a resolution of this ambiguity. Assuming that *some* holographic description exists — regardless of whether it is the NCYM theory — we argue that there should be operators in the holographic boundary theory which create normalized states of definite energy and momenta. An operator version of the dual correspondence then provides a calculation of correlators of these operators in terms of bulk Green's functions. We show that in the low-energy limit the correlators reproduce expected answers of the ordinary Yang-Mills theory.

Keywords: AdS/CFT Correspondance, D-branes.

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1. Introduction

Non-commutative gauge theories appear as limits of D-brane open string theories in the presence of non-vanishing NSNS B-fields [1, 2]. In [3] a precise decoupling limit was defined in which the string tension is scaled to infinity and the closed string metric is scaled to zero keeping the dimensionful NSNS B-field fixed, and it was shown that in this limit the open string theory on D-branes reduces to precisely non-commutative Yang-Mills (NCYM) theory. Furthermore, NCYM appears naturally in the IKKT matrix theory [4], a connection which has led to further insight. Non-commutative field theories have novel perturbative behaviour including an IR/UV mixing [5].

It is natural to ask whether the large-N limit of NCYM is dual to closed string theory in some background, analogous to the duality of usual Yang-Mills theory to string theories [6, 7]. In fact, in [8] and [9] supergravity duals of strong 't Hooft coupling limits of NCYM were proposed. Further evidence for such duality came from the study of D-instantons in such supergravity backgrounds and their relationship to instantons of the NCYM [10]. Aspects of such duality have been explored in [11].

However, the nature of this duality has been rather confusing. In usual AdS/CFT duality, there is a well-defined connection between boundary correlators in supergravity and correlators of local operators of the boundary conformal field theory. Consider AdS_{d+1} with a metric

$$ds^2 = \frac{1}{z^2}\left(-dt^2 + dz^2 + d\vec{x}\cdot d\vec{x}\right), \tag{1.1}$$

where $\vec{x} = x^i, i = 1, \ldots, (d-1)$. The boundary of this space is at $z = z_0$ with $z_0 \ll 1$. In the strong 't Hooft coupling limit the correspondence may be written as

$$e^{-\Gamma_{\text{eff}}[\phi_0^i(x)]} = \int \mathcal{D}A_\mu \, e^{-S_{\text{CFT};z_0} - \sum_i \int dx \phi_0^i(x) \mathcal{O}_i(x)}, \tag{1.2}$$

where $S_{\text{CFT};z_0}$ is the action of a d-dimensional theory with a cutoff z_0. $\Gamma_{\text{eff}}[\phi_0^i(x)]$ is the effective action of supergravity in the background (1.1) as a functional of the values of the various fields $\phi^i(z,x)$ on the boundary,

$$\phi^i(z_0, x) = z_0^{\Delta_i - d} \phi_0^i(x). \tag{1.3}$$

$\mathcal{O}_i(x)$ is the operator in the boundary theory which is dual to the field ϕ^i. This particular behaviour of the fields near the boundary given in (1.3) is guaranteed by the conformal isometries of the AdS space-time and Δ_i turns out to be the conformal dimension of the operator $\mathcal{O}_i(x)$. Thus in this case there is a holographic relationship between the bulk theory and a *local* field theory on the boundary.

In [9] a similar approach was tried for the spacetimes proposed to be duals of NCYM. It was found that to make sense of the results, the relationship between the boundary values of the supergravity fields and the sources of some operators in the proposed dual boundary theory is necessarily nonlocal in position space. In other words the relationship between momentum space quantities involve non-trivial functions of momentum. Similar momentum dependent factors were encountered in the study of one point functions in instanton backgrounds in [10]. It is not *a priori* clear what these factors should be and the entire procedure appears ambiguous since we can get any answer we want by considering different renormalization factors.

In this paper we assume that a holographic description of the bulk theory in such backgrounds exists in terms of a theory living on the boundary — which may or may not be the NCYM theory. This boundary theory will be generically nonlocal and there would not be local operators creating physical states. However, because of translation invariance, there are well defined operators which create *normalized* states of definite energy and momenta. A dual correspondence in this context means that such states are to be identified with states of supergravity modes propagating in this background. Consequently, *bulk* correlation functions in supergravity, with points taken to lie on the boundary, define a set of momentum space boundary correlators unambiguously. This gives a supergravity prediction for such correlators which should reduce to usual Yang-Mills correlators in the low-energy limit. We show that this is indeed true.

One of the motivations to study the dual correspondence in the decoupling limit of [3] is that some of the dual space-times are asymptotically flat in terms of the Einstein metric. Even for the cases where the geometry is not asymptotically flat, we will see that some normal modes of supergravity fluctuations perceive the asymptotic region as flat space-time. The hope is that some understanding of the holographic correspondence may teach us something about holographic relationships in other asymptotically flat spaces. In fact there is a close relationship between the type of problems encountered here and those encountered in the holographic correspondence for NS five branes [12]. Here again the dual spacetimes are asymptotically flat and the boundary theory is nonlocal. Our prescription of reading off correlators from bulk Green's function is in fact closely related to the procedure adopted in [12] where the correlators are identified with relevant S-matrix elements.

In section 2 we present the supergravity solutions we are dealing with and the equations for decoupled modes propagating in this background. In section 3 we calculate the classical action for this on-shell mode as in [9] and state the ambiguity one encounters in trying to extract boundary correlators from this action.[1] In section 4 we give the operator version of the standard AdS/CFT correspondence for $B = 0$ and show how boundary correlators of operators creating normalized states in momentum space can be read off from bulk Green's function. In section 5 we repeat this for our backgrounds with $B \neq 0$ and show how boundary correlators of momentum space operators can be obtained unambiguously. Section 6 contains concluding remarks.

2. The supergravity solution and modes

We will consider two kinds of IIB supergravity backgrounds with non-zero B-fields obtained in [9]. The D3-brane has a worldvolume along $(x^0, \ldots, x^3)$.

The first kind of background has a non-zero NSNS B_{23} with all other components set to zero. In the decoupling limit which corresponds to the low-energy limit of [3] the string metric is

$$ds^2 = \alpha' R^2 \left[u^2 \left(-dx_0^2 + dx_1^2 \right) + \frac{u^2}{1 + a^4 u^4} \left(dx_2^2 + dx_3^2 \right) + \frac{du^2}{u^2} + d\Omega_5^2 \right], \tag{2.1}$$

where the dilaton ϕ, NS field B and the RR 2-form field $\tilde{B}$ and the 5-form field strength F are given by

$$\begin{aligned} e^{2\phi} &= \frac{g^2}{1 + a^4 u^4}, \qquad & B_{23} &= \frac{\alpha' R^2}{a^2} \frac{a^4 u^4}{1 + a^4 u^4}, \\ \tilde{B}_{01} &= \frac{\alpha' a^2 R^2}{g} u^4, \qquad & F_{0123u} &= \frac{\alpha'^2}{g(1 + a^4 u^4)} \partial_u (u^4 R^4), \end{aligned} \tag{2.2}$$

where $R^4 = 4\pi g N$ and g is the *open* string coupling. In the infrared, $u \to 0$ the space time is $AdS_5 \times S^5$.

[1] We correct some errors in the treatment of [9].

In this background, the graviton fluctuation h_{01} with zero momenta along x^0, x^1 and zero angular momenta along the S^5 satisfy a simple decoupled equation. With $\phi = g^{00}h_{01}$ this is

$$\partial_\mu \left(\sqrt{g}\, e^{-2\phi} g^{\mu\nu} \partial_\nu \phi\right) = 0 \tag{2.3}$$

which becomes in terms of modes

$$\phi(u, x_2, x_3) = \int \left[\frac{d^2 k}{(2\pi)^4}\right] \phi\left(\vec{k}, u\right) e^{-ik_2 x^2 - ik_3 x^3}\,, \tag{2.4}$$

$$\partial_u \left(u^5 \partial_u \phi\left(\vec{k}, u\right)\right) - k^2 u\left(1 + a^4 u^4\right) \phi\left(\vec{k}, u\right) = 0\,, \tag{2.5}$$

where in (2.5) $k^2 = k_2^2 + k_3^2$. Such zero energy perturbations do not make sense in lorentzian signature. We will therefore work in the euclidean signature.

For $au \gg 1$, (2.5) becomes the equation for modes of a massless scalar field in flat space. In this sense this mode perceives the asymptotic region as flat.

The second kind of background has self dual B-fields and has in addition a non-trivial axion χ. The decoupling limit solution in euclidean signature is given by the Einstein metric ds_E^2

$$ds_E^2 = \frac{\alpha' R^2}{\sqrt{\hat{g}}} \left[(f(u))^{-1/2} \left(d\tilde{x}_0^2 + \cdots + d\tilde{x}_3^2\right) + (f(u))^{1/2} \left(du^2 + u^2 d\Omega_5^2\right)\right] \tag{2.6}$$

while the other fields are

$$\begin{aligned}
e^{-\phi_0} &= \frac{1}{\hat{g}} - i\,\chi_0 = \frac{1}{\hat{g}}\, u^4\, f(u)\,, \\
\tilde{B}_{01} &= \tilde{B}_{23} = -\frac{i}{\hat{g}}\, B_{01} = -\frac{i}{\hat{g}}\, B_{23} = -\frac{i\alpha' a^2 R^2}{\hat{g}} \left(f(u)\right)^{-1}, \\
F_{0123u} &= \frac{4i(\alpha')^2 R^4}{\hat{g} u^5} \left(f(u)\right)^{-2},
\end{aligned} \tag{2.7}$$

where

$$f(u) = \frac{1}{u^4} + a^4\,, \qquad R^4 = 4\pi \hat{g} N\,. \tag{2.8}$$

An interesting feature of this solution is that the space-time is asymptotically flat (in Einstein metric) even in the decoupling limit. In fact (2.6) is exactly the full D3-brane metric. Furthermore near the boundary $u = \infty$ the string coupling vanishes.

In the background (2.6)–(2.8), it was shown in [10] that there are special fluctuations which satisfy decoupled equations. These are fluctuations of dilaton $\delta\phi$ and axion $\delta\chi$ which obey the condition

$$\delta\chi + ie^{-\phi}\delta\phi = 0\,. \tag{2.9}$$

The corresponding equation for the fluctuation $\delta\phi$ is

$$\nabla_E^2 \left(e^{\phi_0} \delta\phi\right) = 0\,, \tag{2.10}$$

where ∇_E^2 is the laplacian in the Einstein metric ds_E^2 given in (2.6). These fluctuations are now allowed to carry momenta along all the brane worldvolume directions, but for simplicity we will consider zero S^5 angular momenta. The modes are

$$e^{\phi_0}\,\delta\phi(u,x) = \int \left[\frac{d^4k}{(2\pi)^4}\right] \Phi\left(\vec{k},u\right) e^{-i\vec{k}\cdot\vec{x}}\,, \tag{2.11}$$

where

$$\vec{k}\cdot\vec{x} = \sum_{i=0}^{3} k_i\, x^i \tag{2.12}$$

and the equation (2.10) once again becomes

$$\partial_u\left(u^5\partial_u\Phi\left(\vec{k},u\right)\right) - k^2\,u\left(1+a^4u^4\right)\Phi\left(\vec{k},u\right) = 0 \tag{2.13}$$

which is identical to (2.5). In (2.13),

$$k^2 = \sum_{i=0}^{3}(k_i)^2\,. \tag{2.14}$$

3. Solutions and the boundary action

The solutions of eq. (2.13) or (2.5) may be written in terms of Mathieu functions [13]. Introducing the coordinates

$$u = \frac{1}{a}e^{-w} \tag{3.1}$$

the two independent solutions may be chosen to be

$$\frac{1}{u^2}\,H^{(1)}\left(\nu, w+\frac{i\pi}{2}\right), \qquad \frac{1}{u^2}\,H^{(2)}\left(\nu, -w-\frac{i\pi}{2}\right). \tag{3.2}$$

Here the parameter ν is determined in terms of the combination (ka). It has a power series expansion given by

$$\nu = 2 - \frac{i\sqrt{5}}{3}\left(\frac{ka}{2}\right)^4 + \frac{7i}{108\sqrt{5}}\left(\frac{ka}{2}\right)^8 + \cdots. \tag{3.3}$$

The Mathieu functions $H^{(i)}$ have the asymptotic property

$$H^{(i)}(\nu,z) \longrightarrow H_\nu^{(i)}(kae^z)\,, \quad z\to\infty\,, \tag{3.4}$$

where $H_\nu^{(i)}(z)$ denotes Hankel functions. Furthermore, in this region, only $(ka) \ll 1$ contribute significantly, so that $\nu \sim 2$. Thus near $u=0$ where the geometry is $AdS_5\times S^5$ the solutions become the standard solutions of the massless wave equation in AdS, viz $(1/u^2)K_2(k/u)$ and $(1/u^2)I_2(k/u)$.

From the asymptotic behavior of the Hankel functions it is clear that the solution $H^{(1)}(\nu, w + i\pi/2)$ is well behaved and goes to zero in the interior of the spacetime at $u = 0$, while the solution $H^{(2)}(\nu, -w - i\pi/2)$ is well behaved at $u = \infty$.

$$
\begin{aligned}
H^{(1)}\left(\nu, w + \frac{i\pi}{2}\right) &\longrightarrow e^{-i\frac{\pi}{2}(\nu+1)}\sqrt{\frac{2}{\pi k a e^{w}}}\, e^{-kae^{w}}, \quad w \to \infty (u \to 0)\,, \\
H^{(2)}\left(\nu, -w - \frac{i\pi}{2}\right) &\longrightarrow e^{i\frac{\pi}{2}(\nu+1)}\sqrt{\frac{2}{\pi k a e^{-w}}}\, e^{-kae^{-w}}, \quad w \to -\infty (u \to \infty)\,. \quad (3.5)
\end{aligned}
$$

3.1 Supergravity actions and correlators in the usual AdS/CFT correspondence

Let us recall the standard way of obtaining correlators in the AdS/CFT correspondence [7] using (1.2). For a minimally coupled scalar field of mass m in AdS_{d+1} with a metric given by (1.1) one considers a solution (in euclidean signature) which is smooth in the interior

$$
\phi(x, z) = \int \left[\frac{d^d k}{(2\pi)^d}\right] k^{\gamma}\, z^{d/2}\, K_{\gamma}(kz)\, e^{-ik\cdot x}\, \phi_0(k)\,. \tag{3.6}
$$

where $\gamma = \frac{1}{2}\sqrt{d^2 + 4m^2}$. Here x denotes all the four directions $x = (\vec{x}, t)$ and $k^2 = k_0^2 + \vec{k}^2$. One then computes the supergravity action after putting in a boundary at $z = z_0 \ll 1$. Note that the fourier modes $\phi_0(k)$ have been defined so that as one approaches the boundary

$$
\lim_{z\to 0} \phi(z, k) = z^{\frac{d}{2}-\gamma}\phi_0(k) \tag{3.7}
$$

$\phi_0(k)$ are then taken to be sources for operators $O(k)$ conjugate to the supergravity field in the Yang-Mills theory living on the boundary. The action is purely a boundary term with the leading non-analytic piece

$$
S \sim \int \left[\frac{d^d k}{(2\pi)^d}\right] \phi_0(k)\, \phi_0(-k)\, k^{2\gamma} \log(kz_0)\,. \tag{3.8}
$$

The correspondence (1.2) then leads to a two point function of $\mathcal{O}(k)$ with a leading non-analytic piece

$$
\langle \mathcal{O}(k)\mathcal{O}(-k)\rangle \sim k^{2\gamma}\log(kz_0) \tag{3.9}
$$

which shows that the dimension of this operator is

$$
\Delta = \frac{d}{2} + \gamma\,. \tag{3.10}
$$

This specific relation between the sources and the boundary values of the field is simple — the power of the infrared cutoff which appears in (3.7) is $(d - \Delta)$. By the IR/UV correspondence this is precisely the power of *ultraviolet* cutoff necessary to add a perturbation $\mathcal{O}(k)$ to the boundary action.

3.2 Supergravity actions for $B \neq 0$

The proposal of [9] is to consider $u = \infty$ as the boundary also for the classical solutions in the presence of a B-field. The solution to be used in calculating the classical action has to be regular in the interior which means we have to take

$$\phi_k(u) = \frac{1}{u^2} H^{(1)}\left(\nu, w + \frac{i\pi}{2}\right) e^{i\frac{\pi}{2}(\nu+1)}\, \phi_0(k)\,. \tag{3.11}$$

Once again the classical action is a boundary term. To evaluate this term we need to find the behavior of solution (3.11) for large values of u. This can be done using the relation

$$\begin{aligned} H^{(1)}(\nu, w) = \frac{1}{C(ka)} &\left[\left(\chi(ka) - \frac{1}{\chi(ka)}\right) H^{(1)}(\nu, -w) + \right. \\ &\left. + \left(\chi(ka) - \frac{1}{\eta(ka)^2 \chi(ka)}\right) H^{(2)}(\nu, -w)\right], \end{aligned} \tag{3.12}$$

where we have defined

$$\eta(ka) = e^{i\pi\nu}\,, \qquad C(ka) = \eta(ka) - \frac{1}{\eta(ka)} \tag{3.13}$$

and the function $\chi(ka)$ has been defined in [13] in terms of relations between various Mathieu functions. Defining further the functions

$$A(ka) = \chi(ka) - \frac{1}{\chi(ka)}\,, \qquad B(ka) = \eta(ka)\chi(ka) - \frac{1}{\eta(ka)\chi(ka)} \tag{3.14}$$

the asymptotic form of the solution $\phi_k(u)$ becomes

$$\phi_k(u) \longrightarrow \frac{1}{u^2 C(ka)} \sqrt{\frac{2}{\pi k a^2 u}} \left[i A(ka) e^{ka^2 u} - \hat{B}(ka) e^{-ka^2 u} \right] \phi_0(k)\,, \quad (u \to \infty)\,, \tag{3.15}$$

where $\hat{B}(ka)$ denotes the real part of $B(ka)$.[2] We will see shortly that $A(ka)$ is purely imaginary so that the expression in (3.15) is real. As expected the solution diverges exponentially at the boundary $u = \Lambda$ with $\Lambda \gg 1$. The contribution to the classical action from this solution, which becomes the boundary term

$$\left[u^5 \phi_{-k}(u) \partial_u \phi_k(u)\right]_{u=\Lambda} \tag{3.16}$$

is clearly divergent. Subtracting this infinite piece we get a term where the exponentials cancel leaving a contribution

$$S_B = \frac{5}{2\Lambda} \frac{2}{\pi k a^2} \frac{i A(ka) \hat{B}(ka)}{C^2(ka)}\, \phi_0(k)\, \phi_0(-k)\,. \tag{3.17}$$

Note that the boundary action given in [9] has an error and differs from the above by essentially a factor of $1/\Lambda$.

[2] The real part has to be taken in this asymptotic expansion since the wave functions are real. This is similar to what happens in the asymptotic expansions of modified Bessel's functions.

Mimicking the standard procedure in the AdS/CFT correspondence we might want to relate the functions $\phi_0(k)$ to source terms in the dual theory living on the boundary, so that derivatives of the classical action with respect to these would give correlation functions of the dual operators. So far, however, we have no clue about this precise relationship. The only guide we have at this stage is that in the low-energy limit $ka \ll 1$ the correlators should reproduce the known answers in the $AdS_5 \times S^5$ case — for these minimally coupled scalars the two point function should go as $k^4 \log(k)$.

3.3 Low-energy limits

We need to find the low ka expansion of the various functions. This may be done using the results of [13] as follows. First note that the function $\eta(ka)$ is purely real, as follows from the expression for ν in (3.3). The functions $A(ka), B(ka)$ and $C(ka)$ satisfy a unitarity relation [13]

$$|B(ka)|^2 = |A(ka)|^2 + |C(ka)|^2 . \tag{3.18}$$

Using the reality of $\eta(ka)$ and hence C it may be easily shown from (3.18) that $\chi(ka)$ must be a pure phase, so that $A(ka)$ is purely imaginary. Denoting

$$\eta(ka) = e^{\beta(ka)} , \qquad \chi(ka) = e^{i\gamma(ka)} \tag{3.19}$$

and using (3.18) the various functions may be expressed in terms of the function

$$P(ka) = \frac{|C(ka)|^2}{|B(ka)|^2} \tag{3.20}$$

which is the absorption probability in the full D3-brane background computed in [13]. We give below some expressions which we will need

$$\begin{aligned} \frac{\hat{B}(ka)}{iA(ka)} &= \left[\frac{P(ka)\cosh^2\beta(ka) - \sinh^2\beta(ka)}{1 - P(ka)}\right]^{1/2} , \\ \frac{iA(ka)}{C(ka)} &= \left[\frac{1}{P(ka)} - 1\right]^{1/2} . \end{aligned} \tag{3.21}$$

The expansion of $P(ka)$ given in [13] is

$$P(ka) = A_0(ka)^8 \Big[1 + A_1(ka)^4 + A_2(ka)^4 \log(ka) + \cdots\Big] , \tag{3.22}$$

where A_i are numerical constants. The expansion of $\beta(ka)$ is of the form

$$\beta(ka) = \beta_0(ka)^4 \Big[1 + \beta_1(ka)^4 + \beta_2(ka)^8 + \cdots\Big] , \tag{3.23}$$

where β_i are numerical coefficients. This leads to the following expansions

$$\begin{aligned}\frac{\hat{B}(ka)}{iA(ka)} &\sim (ka)^4\Big[1+\alpha_1(ka)^4+\alpha_2(ka)^4\log(ka)+\cdots\Big]\,,\\ \frac{iA}{C(ka)} &\sim \frac{1}{(ka)^4}\Big[1+\gamma_1(ka)^4+\gamma_2(ka)^4\log(ka)+\cdots\Big]\,,\end{aligned}\tag{3.24}$$

where the coefficients α_i and γ_i may be obtained from the expansions in (3.22) and (3.23). Using these, the action S_B is seen to behave as

$$S_B \sim \frac{1}{a(ka)^5}\Big[1+(ka)^4+(ka)^4\log(ka)\Big]\frac{1}{\Lambda}\phi_0(k)\phi_0(-k)\tag{3.25}$$

for small momenta. Therefore if we define a renormalized boundary value of the field

$$\Phi_0(k) = F(ka)\phi_0(k)\tag{3.26}$$

such that

$$F(ka) \sim \frac{1}{(ka)^{5/2}}\frac{1}{\Lambda^{1/2}}\tag{3.27}$$

for small momenta, and declare that $\Phi_0(k)$ are the sources which couple to the boundary theory operator dual to the bulk field ϕ, we would certainly get the correct low momentum behavior for the non-analytic piece of the two point function.[3]

Clearly, this is an arbitrary procedure. Once we use momentum dependent factors to renormalize fields, we can get any answer we want! At this stage there is no obvious principle which determines this factor. Any statement about holography would be an empty statement.

4. Bulk and boundary Green's functions in AdS/CFT

The usual AdS_{d+1}/CFT_d correspondence may be understood in terms of the modes of bulk field operators. Our treatment follows [14] with some differences. Consider quantization of a massive scalar field $\phi(z,\vec{x},t)$ of mass m which is minimally coupled to the AdS metric (1.1). The field has the following mode expansion

$$\begin{aligned}\phi(z,\vec{x},t) = \frac{z^{d/2}}{2R^{(d-1)/2}}\int_0^\infty d\alpha \int \frac{d^{d-1}k}{(2\pi)^d}\left(\frac{\alpha}{\omega}\right)^{1/2} J_\gamma(\alpha z)\,\times\\ \times\left[a\left(\vec{k},\alpha\right)e^{-i(\omega t-\vec{k}\cdot\vec{x})}+(\text{h.c.})\right],\end{aligned}\tag{4.1}$$

where

$$\omega^2 = \vec{k}^2+\alpha^2\,,\qquad \gamma = \frac{1}{2}(d^2+4m^2)^{1/2}\,.\tag{4.2}$$

[3] In [9] it is claimed that the two point function of the operators defined there (which differs from ours) has the correct low-energy behavior. We have not been able to see how this follows.

The modes are normalized so that the annihilation/creation operators satisfy the standard commutators

$$\left[a\left(\vec{k},\alpha\right), a^\dagger\left(\vec{k}',\alpha'\right)\right] = \delta^{(d-1)}\left(\vec{k}-\vec{k}'\right)\delta(\alpha-\alpha')\,. \tag{4.3}$$

We can now make a change of integration variables from $(\alpha,\vec{k})$ to $(\omega,\vec{k})$ and define new operators[4]

$$b\left(\vec{k},\omega\right) = \left(\frac{\omega}{\alpha}\right)^{1/2} a\left(\vec{k},\alpha\right) \tag{4.4}$$

which satisfy the commutation relations

$$\left[b\left(\vec{k},\omega\right), b^\dagger\left(\vec{k}',\omega'\right)\right] = \delta^{(d-1)}\left(\vec{k}-\vec{k}'\right)\delta(\omega-\omega') \tag{4.5}$$

and rewrite the expansion (4.1) as

$$\phi(z,\vec{x},t) = \frac{z^{d/2}}{2R^{(d-1)/2}}\int d\omega \int \frac{d^{d-1}k}{(2\pi)^d}\, J_\gamma(\alpha z)\left[b\left(\vec{k},\omega\right)e^{-i(\omega t-\vec{k}\cdot\vec{x})} + (\mathrm{h.c.})\right]. \tag{4.6}$$

In (4.6) α is determined in terms of ω and $\vec{k}$ by the relation (4.2). The states created by $b^\dagger(\omega,\vec{k})$, denoted as $|\omega,\vec{k}\,\rangle = b^\dagger(\omega,\vec{k})|0\,\rangle$ are normalized according to d-dimensional delta functions, as follows from (4.5),

$$\left\langle \omega,\vec{k}\middle|\omega',\vec{k}'\right\rangle = \delta^{(d-1)}\left(\vec{k}-\vec{k}'\right)\delta(\omega-\omega')\,. \tag{4.7}$$

The holographic correspondence then implies that these states are also states in the d-dimensional boundary theory and there are composite operators which create these states. We can define these operators in momentum space as

$$\mathcal{O}\left(\omega,\vec{k}\right) = \frac{2\pi}{R^{(d-1)/2}}\left(\omega^2-\vec{k}^2\right)^{\gamma/2}\left[\theta(\omega)b(\omega,k) + \theta(-\omega)b^\dagger\left(-\omega,-\vec{k}\right)\right]. \tag{4.8}$$

The overall power of $\alpha = (\omega^2-\vec{k}^2)^{1/2}$ follows from the fact that as we approach the boundary $z\to 0$ the radial wave function $J_\gamma(\alpha z)\to(\alpha z)^\gamma$. This allows us to define in an unambiguous way a momentum space correlation function of the boundary theory in terms of the boundary values of the fourier transform of the Feynman Green's function of the bulk field

$$\left\langle \mathcal{O}\left(\omega,\vec{k}\right)\mathcal{O}\left(-\omega,-\vec{k}\right)\right\rangle = \lim_{z,z'\to 0}\left(\frac{\alpha^{2\gamma}}{\psi_{\omega,\vec{k}}(z)\psi_{-\omega,-\vec{k}}(z')}\right)G_F(z,z';\omega,k)\,, \tag{4.9}$$

where we have used translation invariance along the $\vec{x}$ directions and $\psi_{\omega,\vec{k}}(z)$ denote the bulk radial wavefunctions

$$\psi_{\omega,\vec{k}}(z) = z^{d/2}J_\gamma(\alpha z)\,. \tag{4.10}$$

[4]There is a subtely here. In lorentzian signature we must have $\omega^2 > k^2$ so that the range of integration over the four momenta is strictly restricted. However, this fact has no consequence for two point functions. We thank E. Martinec for discussions about this point.

The corresponding wavefunctions $\psi_{\omega,\vec{k}}(t,\vec{x},z) = z^{d/2} J_\gamma(\alpha z)\, e^{-i(\omega t - \vec{k}\cdot\vec{x})}$ are normalized in terms of the standard Klein-Gordon norm

$$\left(\psi_{\omega,\vec{k}}(\vec{x},t,z), \psi_{\omega',\vec{k}'}(\vec{x},t,z)\right) = \delta(\omega-\omega')\delta\left(\vec{k}-\vec{k}'\right). \tag{4.11}$$

We can now consider a Wick rotation to obtain a relation between euclidean Green's functions

$$\langle \mathcal{O}(k)\mathcal{O}(-k)\rangle_E = \lim_{z,z'\to 0}\left(\frac{k^{2\gamma}}{\psi_k(z)\psi_{-k}(z')}\right) G_E(z,z';\omega,k)\,, \tag{4.12}$$

where the wave functions are also rotated to euclidean space and k without a vector sign denotes the d-dimensional euclidean momenta.

It may be easily checked that the euclidean bulk Green's function leads to the euclidean correlators on the boundary obtained according to the procedure of [7]. The euclidean bulk Green's function is

$$G_E(z,k;z',-k) = (zz')^{d/2} K_\gamma(kz) I_\gamma(kz')\,, \quad z' < z\,. \tag{4.13}$$

Using the asymptotic form of the modified Bessel functions for $z, z' \to 0$, we easily see that the leading non-analytic piece of the Green's function is

$$G_E(z,z',k) \longrightarrow (zz')^{\gamma+d/2}\, k^{2\gamma} \log(kz)\,. \tag{4.14}$$

Using (4.12) we get the boundary correlator

$$\langle \mathcal{O}(k)\mathcal{O}(-k)\rangle \sim k^{2\gamma}\log(kz) \tag{4.15}$$

in agreement with (3.9).

We can in fact define *local* operators on the boundary by taking the fourier transform of the momentum space field $\mathcal{O}(\omega,\vec{k})$. In fact the power of α in (4.8) has been chosen such that this boundary field is in fact the boundary value of the bulk field upto the value of the bulk wavefunction at the boundary

$$\lim_{z\to 0}\phi(z,\vec{x},t) = (z)^{\gamma+d/2}\mathcal{O}(\vec{x},t) \tag{4.16}$$

thus defining $\mathcal{O}(\vec{x},t)$. The reason why this can be done in an unambiguous fashion is that the wavefunctions decay as the *same* power of z regardless of the value of the momenta. This in turn is a consequence of the conformal isometries of AdS space. For IIB supergravity on $AdS_5 \times S^5$ it is known that the boundary operators thus defined are in fact *local* gauge invariant operators of $N = 4$ SYM theory.

5. Bulk and boundary Green's functions for $B \neq 0$

For our supergravity backgrounds with $B \neq 0$ the conjectured dual theory — NCYM — is not a local quantum field theory in the conventional sense. The backgrounds we are dealing with are not asymptotically AdS — in fact the second class of euclidean backgrounds are asymptotically flat. However we do have translation invariance along the brane directions — so that physical states of the NCYM can be still labelled by the energy and momenta. If this duality is indeed correct, we should be able to represent such states by on shell states of supergravity with the same values of energy and momentum — essentially repeating the treatment of section 2. In this section we argue that such a correspondence between operators in momentum space leads to an unambiguous supergravity prediction of two point functions for the corresponding operators in the NCYM theory. We will then verify that this has the correct low-energy behavior.

For the kind of fields which satisfy the minimally coupled massless Klein Gordon equation as we have been studying the mode expansion in our background may be written down in analogy to (2.6) with the Bessel function being replaced by the appropriate Mathieu function. With lorentzian signature there are two independent wavefunctions which are (delta function) normalizable, corresponding to incoming and outgoing waves in the full D3-brane geometry. In the following we will consider only the wave function whose euclidean continuation does not blow up at infinity — this is the wave function given by

$$\Psi_k(u) = N(ka)\, e^{-i\frac{\pi}{2}(\nu+1)}\, \frac{1}{u^2}\, H^{(2)}(\nu, -w)\,, \tag{5.1}$$

where $N(ka)$ is a normalization factor. The mode expansion may be then written as

$$\phi(\vec{x}, u, t) = \int \frac{d^3k}{(2\pi)^3}\frac{d\omega}{2\pi} N(ka)\frac{1}{u^2} H^{(2)}(\nu, -w) e^{-i\pi\nu/2}\left[e^{-i(\omega t - \vec{k}\cdot\vec{x})}\hat{a}(\omega, k) + \text{c.c.}\right]. \tag{5.2}$$

$N(ka)$ is determined by requiring that the operators $\hat{a}(\omega, k)$ satisfy the standard commutation relation

$$\left[\hat{a}(\omega, k), \hat{a}^\dagger(\omega', k')\right] = \delta^{(3)}\left(\vec{k} - \vec{k}'\right)\delta(\omega - \omega')\,. \tag{5.3}$$

Of course there is no ambiguity in this expansion. Following the logic of section 2 we then conclude that there are operators which create well defined states and the two point function of these operators are related to the bulk Green's functions by a relation similar to (4.12)

$$\langle \mathcal{O}(k)\mathcal{O}(-k)\rangle_E = \lim_{u,u'\to\infty}\left(\frac{k^4}{\Psi_k(u)\Psi_{-k}(u')}\right)\mathcal{G}_E(u, u'; k)\,, \tag{5.4}$$

where $\mathcal{G}_E$ is the euclidean Green's function in this background. (In this expression k stands for the four vector.)

The Green's function $\mathcal{G}_E$ can be easily computed since we know the two independent classical solutions — in fact this has already been computed in [10] to obtain the D-instanton solution in these backgrounds. This is given by

$$\mathcal{G}_E(u,u';k) = \frac{\pi C(ka)}{4i(uu')^2 A(ka)} H^{(1)}\left(\nu, w+\frac{i\pi}{2}\right) H^{(2)}\left(\nu, -w'-\frac{i\pi}{2}\right), \quad u'>u\,,$$
$$\mathcal{G}_E(u,u';k) = \frac{\pi C(ka)}{4i(uu')^2 A(ka)} H^{(1)}\left(\nu, w'+\frac{i\pi}{2}\right) \times$$
$$\times H^{(2)}\left(\nu, -w-\frac{i\pi}{2}\right), \quad u>u'\,. \tag{5.5}$$

Using the asymptotic expressions for the Mathieu functions (equation (3.15)) one gets for $u' > u \gg 1$

$$\mathcal{G}_E(u,u';k) = \mathcal{G}_0(u,u';k) + \frac{1}{2ka^2}\frac{1}{(uu')^{5/2}}\frac{i\hat{B}(ka)}{A(ka)}\, e^{-ka^2(u+u')}\,, \tag{5.6}$$

where $\mathcal{G}_0$ denotes the Green's function in *flat* space

$$\mathcal{G}_0(u,u';k) = \frac{1}{2ka^2}\frac{1}{(uu')^{5/2}} e^{-ka^2(u'-u)}\,. \tag{5.7}$$

In (5.4) we will have to substitute this asymptotic $\mathcal{G}_E$. In view of the fact that the Green's function became a sum of the free piece and a "connected" piece suggests that it is natural to subtract out the free piece in (5.4). We will adopt this prescription.

Note that the expression (5.4) is unambiguous — there is no room for momentum dependent wave function renormalizations here. The Green's function $\mathcal{G}_E$ is completely determined once the wave equation is known, including all normalizations and the wave functions are determined upto phases which in any case cancel in this expression.

If (5.4) is to make any sense, the u dependence should cancel in the right hand side. From the asymptotic form of $H^{(2)}$ given in (3.5) and the form of the connected bulk Green's function in (5.6) we see that this indeed happens. The final answer is

$$\langle \mathcal{O}(k)\mathcal{O}(-k)\rangle = k^4 \frac{i\hat{B}(ka)}{A(ka)}\left(\frac{1}{N(ka)}\right)^2. \tag{5.8}$$

We do not know how to obtain an explicit expression for $N(ka)$ for all ka — though we emphasize that it can be obtained in principle. However it is straightforward to find the behavior of this normalization factor for small values of ka. This is because in this regime the various Mathieu functions become Bessel functions. Using the relation (notations are those of [13])

$$H^{(2)}(\nu,-z) = \frac{\eta(ka)}{\chi(ka)} H^{(1)}(\nu,z) - \frac{2A(ka)}{C(ka)} J(\nu,z) \tag{5.9}$$

we see that in this regime

$$H^{(2)}\left(\nu, -w-\frac{i\pi}{2}\right) \longrightarrow \frac{2iA(ka)}{C(ka)} e^{i\frac{\pi}{2}(\nu+1)} I_\nu\left(\frac{k}{u}\right). \tag{5.10}$$

Since I_ν is the euclidean continuation of the normalizable solution in the AdS limit we now know that for $ka \ll 1$ we must have

$$N(ka) \longrightarrow \frac{C(ka)}{iA(ka)}\,. \tag{5.11}$$

Plugging in the small ka expansions for the various functions, given in (3.24) we easily verify that the leading non-analytic term for small ka is given by

$$\langle \mathcal{O}(k)\mathcal{O}(-k)\rangle \sim k^4 \log(ka) \tag{5.12}$$

which is the answer in the absence of a B-field.

The fact that we have obtained the correct low (ka) behavior is non trivial. This is because we have taken the boundary at $u = \infty$ *before taking any low-energy limit.* We consider this result to be an evidence in favor of the holographic correspondence proposed in [9].

The procedure adopted above to obtain the correlation functions is in fact similar to the procedure adopted in [12] for the case of NS five branes where again one has an asymptotically flat geometry in the decoupling limit. In this work the two point function is related to the S-matrix. This can be of course obtained from the euclidean Green's function using a reduction procedure. Our procedure is of course valid for the standard AdS case where the space-time is not asymptotically flat.

6. Conclusions

The boundary theory proposed to be dual to the supergravity backgrounds we have considered in this paper — viz. NCYM theory — is nonlocal. It is likely that there are no local gauge invariant operators (in terms of the usual non-commutative gauge fields) in this theory and correlation functions in position space do not make any obvious sense. In this paper we have argued that whether or not the boundary theory is indeed NCYM theory, states are still specified by values of the energy and (three-dimensional) momenta. Thus there must be *some* operators $\mathcal{O}(k)$ in momentum space which create such states normalized in the standard fashion. The holographic correspondence identifies these states with normalized states in supergravity. We have shown that this implies an unambiguous prediction of the correlators of $\mathcal{O}(k)$. We verified that for small momenta they reproduce the expected result for usual YM theory.

If the boundary theory is indeed the NCYM theory, it is crucial to find the momentum space dual operators. In the standard AdS/CFT correspondence one useful way to read off the dual operators is to consider the coupling of a three brane to background supergravity fields, using, e.g. a non-abelian version of the DBI-WZ action [15]. One possibility is to explore these couplings in the presence of a B-field and in the low-energy limit of [3]. Rewriting DBI-WZ actions will not be sufficient in this case since the non-trivial modifications come when the supergravity backgrounds carry momenta in the brane direction so that derivatives of gauge fields are important. Some couplings of closed string modes to open string modes in the presence of B-field have been studied in [16]. However it remains to be seen whether one could extract the couplings in terms of the standard fields of the NCYM theory. In particular it appears that the effect of B-fields may not be all encoded in the star product once closed string couplings are included.

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Note Added. While the paper was being typed, a related paper appeared on the net [17] which has some overlap with our work.

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